I. *An Inquiry concerning the* Figure of ſuch Planets as revolve about an Axis, *ſuppoſing the Denſity continually to vary, from the Centre towards the Surface; by Mr.* Alexis Clairaut, *F. R. S. and Member of the Royal Academy of Sciences at* Paris. *Tranſlated from the* French *by the Rev.* John Colſon *Lucas. Prof. Math.* Cantab. *and F. R. S.*

NOtwithſtanding that Part of Sir *Iſaac Newton*'s *Mathematical Principles of Natural Philoſophy*, where he treats of the Figure of the Earth, is deliver'd with the uſual Skill and Accuracy of that great Author; yet I thought ſomething farther might be done in this Matter, and that new Inquiries may be propoſed, which are of no ſmall Importance, and which poſſibly he overlook'd, through the Abundance of thoſe fine Diſcoveries he was in Purſuit of.

What at firſt ſeem'd to me worth examining, when I apply'd myſelf to this Subject, was to know why Sir *Iſaac* aſſumed the Conical Ellipſis for the Figure of the Earth, when he was to determine its Axis? For he does not acquaint us why he did it, neither can we perceive how he had ſatisfied himſelf in this Particular: And unleſs we know this, I think we cannot intirely acquieſce in his Determinations of the Axes of the Planets. It ſeems as if he might have took any other oval Curve, as well as the conical Ellipſis of *Apollonius*, and then he would have come to other Concluſions about thoſe Axes.

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I began then with convincing myſelf by Calculation, that the Meridian of the Earth, and of the other Planets, is a Curve very nearly approaching to an Ellipſis; ſo that no ſenſible Error could enſue by ſuppoſing it really ſuch. I had the Honour of communicating my Demonſtration of this to the ROYAL SOCIETY, at the Beginning of the laſt Year; and I have ſince been inform'd, that Mr. *Stirling*, one of the greateſt Geometricians I know in *Europe*, had inſerted a Diſcourſe in the *Philoſophical Tranſactions*, N° 438. wherein he had found the ſame thing before me, but without giving his Demonſtration. When I ſent that Paper to *London*, I was in *Lapland*, within the frigid *Zone*, where I could have no Recourſe to Mr. *Stirling*'s Diſcourſe, ſo that I could not take any Notice of it.

The Elliptical Form of the Meridian being once proved, I no longer found any thing in Sir *Iſaac Newton*, about the Figure of the Earth, which could create any new Difficulty; and I ſhould have thought this Queſtion ſufficiently diſcuſs'd, if the Obſervations made under the Arctick Circle had not prevail'd on us to believe, that the Shape of the Earth was ſtill flatter than that of Sir *Iſaac*'s Spheroid; and if he himſelf had not pointed at the Cauſes, which might make *Jupiter* not quite ſo flat, as by his Theory, and the Earth ſomething more.

As to *Jupiter*, he ſays, (*Page* 416 of the 3d Edition of *Phil. Nat. Prin. Math.*) that its Equator conſiſts of denſer Parts than the reſt of its Body, becauſe its Moiſture is more dried up by the Heat of the Sun. But as to the Earth, he ſuſpects its Flatneſs to be a ſmall matter greater than what ariſes by his Calculation.

lation. He inſinuates, that it may poſſibly be more denſe towards the Centre than at the Superficies. (*See* Art. XXIV. following) I am ſomething ſurprized that Sir *Iſaac* ſhould imagine, that the Sun's Heat can be ſo great at *Jupiter*'s Equator, when it has no ſuch Effect at that of the Earth; and that he does not aſcribe each to a like Cauſe, by ſuppoſing alſo, that *Jupiter* may be of a different Denſity at the Centre from that at the Superficies.

But whatever Reaſon he might have for introducing two different Cauſes, I give the Preference to the Hypotheſis which ſuppoſes unequal Denſities at the Centre and at the Circumference. I have inquired, by the Aſſiſtance of this Theory, what would be the Figure of the Earth, and of the other Planets which revolve about an Axe, on Suppoſition that they are compoſed of ſimilar *Strata*, or Layers, at the Surface; but that their variable Denſity, from the Centre towards the Circumference, may be expounded by any Algebraical Equation whatſoever.

And though my Hypotheſis ſhould not be conformable to the Laws of Nature, or even though it ſhould be of no real Uſe (which would be the Caſe, if the Obſervations made by the Mathematicians now in *Peru*, compared with ours in the North, ſhould require that Proportion of the Axes, which is derived from Sir *Iſaac*'s Spheroid); I thought however that Geometricians would be pleaſed with the Speculations contain'd in this Paper, as being, if not uſeful, yet curious Problems at leaſt.

PART I.

In which are found the Laws of Attraction, which are exerted upon Bodies at a Diſtance, by a Spheroid compos'd of Orbs of different Degrees of Denſity.

PROBLEM I.

To find the Attraction which a homogeneous Spheroid BNEbe, (TAB. 1. Fig. 1.) *differing but very little from a Sphere, exerts upon a Corpuſcle placed at* A *in the Axis of Revolution.*

I. We may conceive the Space BNEbDMB, included between the Spheroid and the Sphere, to be divided into an infinite Number of Sections perpendicular to the Axe ACb. Suppoſing then that every one of the Particles, which are contain'd in one of theſe Elements or Moments NnmM, exerts the ſame Quantity of Attraction upon the Body at A, which may be ſuppos'd becauſe of the Smallneſs of NM; we ſhall have $c\alpha \times PM^2 \times Pp \times \frac{AP}{AM^3}$ for the Attraction of any one of theſe Elements; putting c for the Ratio of the Circumference to the Radius, and α for the given Ratio of MN to PM, that is, of DE to CD.

Now if we make $CA = e$, $CB = r$, $AM = z$; and for PM, AP, Pp, if we ſubſtitute their Values expreſs'd by z, and then ſeek the Fluent of the foregoing Quantity; we ſhall have $\frac{4c\alpha r^3}{3ee} - \frac{4c\alpha r^5}{5e^4}$ for the Value

Value of the whole Attraction of the Solid generated by the Revolution of BDbEB: To which if we add $\frac{2r^3c}{3ee}$, the Attraction of the Sphere, we ſhall have $\frac{2r^3c}{3ee}+\frac{4cr^3\alpha}{3ee}-\frac{4c\alpha r^5}{5e^4}$ for the required Attraction of the Spheroid upon the Corpuſcle A.

PROBLEM II.

Suppoſing now the Spheroid Bebe (Fig. 2.) *to be no longer of a homogeneous Matter, but to be compoſ'd of an infinite Number of Elliptical* Strata, *all ſimilar to* BEb, *the Denſities of which are repreſented by the Ordinates* KT *of any Curve whatever* VT, *of which we have the Equation between* CK *and* KT; *the Attraction is required which this Spheroid exerts upon a Corpuſcle placed at the Pole* B.

II. Making BC=e, CK=r, by the foregoing Propoſition, we ſhould have $\frac{2r^3c}{3ee}+\frac{4cr^3\alpha}{3ee}-\frac{4c\alpha r^5}{5e^4}$ for the Attraction of the Spheroid KLK, if it conſiſted of homogeneous Matter; and the Fluxion of this Quantity $\frac{2rrc\dot{r}}{ee}+\frac{4c\alpha r^2\dot{r}}{ee}-\frac{4c\alpha r^4\dot{r}}{e^4}$ would be the Element or Moment of the Orb KLKklk. But becauſe the Denſity is variable, we muſt multiply this Value of the Attraction of the Orb by KT, and the Fluent of this Quantity will be the Value of the Attraction of the Spheroid KLK.

As to the Value of KT, which expreſſes the Denſity of the Stratum or Bed KLKklk, we ſhall take only

only fr^p+gr^q, becauſe we ſhall ſee afterwards, that a Value more compounded, at $fr^p+gr^q+hr^s+ir^t$, *&c.* which by the Property of Series may expreſs all Curves, would not produce any Variety in the Calculation.

Therefore multiplying the foregoing Equation by fr^p+gr^q, we ſhall have $\frac{2cf\times\overline{1+2\alpha}\times r^{\overline{3+p}}}{ee\times\overline{3+p}}-\frac{4c\alpha fr^{\overline{5+p}}}{e^4\times\overline{5+p}}+\frac{2cg\times\overline{1+2\alpha}\times r^{\overline{3+q}}}{ee\times\overline{3+q}}-\frac{4c\alpha gr^{\overline{5+q}}}{e^4\times\overline{5+q}}$ for the Quantity of Attraction of the Spheroid KLK, exerted upon a Corpuſcle placed at B.

III. In this Value making $r=e$, we ſhall have $\frac{2cfe^{\overline{1+p}}}{3+p}+\frac{8cfe^{\overline{1+p}}\alpha}{\overline{3+p}\times\overline{5+p}}+\frac{2cge^{\overline{1+q}}}{3+q}+\frac{8cge^{\overline{1+q}}\alpha}{\overline{3+q}\times\overline{5+q}}$, which will expreſs the Force of Attraction of the Spheroid BEb, exerted upon a Corpuſcle placed at the Pole B.

THEOREM.

A Corpuſcle being placed in any Point N *of the Surface of the foregoing Spheroid* BEbe, *I ſay it will undergo the ſame Attraction from this Spheroid, as if it were placed at the Pole* N *of a ſecond Spheroid revolving about the Axe* NO, *the ſecond Axe being the Radius of a Circle equal in Superficies to the Ellipſis* FG; *ſuppoſing this ſecond Spheroid* NGOF (Fig. 3.) *to be compoſed of the* Strata MmqQ, *whoſe Denſities are the ſame as thoſe of the* Strata KkLlKk, *of the firſt Spheroid.*

IV. In

IV. In the Diſcourſe I had the Honour of communicating to the ROYAL SOCIETY, being then at *Torneo*, printed in the *Philoſophical Tranſactions*; N° 445. I have demonſtrated this Propoſition as to a homogeneous Spheroid; and the ſame Reaſoning will obtain in this Caſe alſo.

PROBLEM III.

To find the Attraction which the Spheroid Bebe *(Fig. 2.) exerts upon a Corpuſcle placed at any point* N *of the Superficies.*

V. We will make, as above, $BC = e$, $CE = e + e\alpha$, and alſo $CN = e + e\lambda$, and half the Conjugate Diameter of CN will be $CG = e + e\alpha - e\lambda$; whence the Radius of a Circle, equal in Superficies to the Ellipſis EG, will be a mean proportional between CE and and CG, that is to ſay, $e + e\alpha - \frac{1}{2}e\lambda$. Therefore the Spheroid BEbe exerts the ſame Attraction at N, as would be exerted at the Pole of a Spheroid NGOF, (Fig. 3.) of which the principal Axis would be $NO = 2e + 2e\lambda$, and the ſecond would be to the Principal as $1 + \alpha - \frac{3}{2}\lambda$ to 1.

Therefore in the Expreſſion of the Attraction at the Pole, (Art. III.) we muſt ſubſtitute $e + e\lambda$ inſtead of e, and $\alpha - \frac{3}{2}\lambda$ inſtead of α. But if f and g muſt no longer be the ſame; for we may eaſily perceive by the foregoing *Theorem*, that the Denſity muſt be the ſame in this Spheroid NGOF, at the Diſtance $r + r\lambda$ from the Centre, as it is in the Spheroid BEbe at the

Diſtance r. Therefore $f\left(\frac{e}{1+\lambda}\right)^{p}+g\left(\frac{e}{1+\lambda}\right)^{q}$ muſt be put inſtead of $fe^{p}+ge^{q}$. Thus we ſhall have $\frac{2cfe^{1+p}}{3+p}+\frac{\overline{2p-2}cf\lambda e^{1+p}}{\overline{3+p}\times\overline{5+p}}+\frac{8cf\alpha e^{1+p}}{\overline{3+p}\times\overline{5+p}}+\frac{2cge^{1+q}}{3+q}$ $+\frac{\overline{2q-2}cg\lambda e^{1+q}}{\overline{3+q}\times\overline{5+q}}+\frac{8cg\alpha e^{1+q}}{\overline{3+q}\times\overline{5+q}}$ for the Attraction of the Spheroid BE be at N.

VI. If we make $\lambda=\alpha$, the foregoing Expreſſion will be reduced to this $\frac{2cfe^{1+p}}{3+p}+\frac{2cfe^{1+p}\alpha}{5+p}$ $+\frac{2cge^{1+q}}{3+q}+\frac{2cge^{1+q}\alpha}{5+q}$, which expreſſes the Attraction of the Equator.

VII. If we would have the Attraction at any Point M within the Spheroid, in the Expreſſion of the Attraction at N, we muſt put r inſtead of e. The Proof of this is plain from the ſame Reaſons that Sir *Iſaac Newton* makes uſe of, (*Corol.* 3. *Prop.* XCI. *L.* 1. *Princip. Math.*) to ſhew that the Attraction of an Elliptic Orb, at a Point within it, is none at all.

PROBLEM IV.

Let RΠrπ (Fig. 4.) *be a Circle whoſe Centre is* Y; *'tis required to find the Attraction which this Circle exerts upon a Corpuſcle at* N, *according to the Direction* HY; *ſuppoſing the Point* H, *which anſwers perpendicularly below the Point* N, *to be at a very ſmall Diſtance from the Point* Y.

VIII. Let there be drawn ΠHπ perpendicular to the Diameter RYr, and let the Space RΠπ be tranſ-

ferr'd

ſer'd to $\pi\Pi Z$. Then the Space $\pi Z\Pi r$ will be the only Part of $R\Pi r\pi$, which will attract the Body N according to HY.

To find the Attraction of this little Space, we will ſuppoſe it to be divided into the Elements TtsS, the Attractions of which, according to HY, will be $\frac{TtsS\times QT}{NT^3}$, or $\frac{2HY\times Qq\times QT}{NT^3}$, the Fluent of which $\frac{2HY\times HQTZ}{NT^3}$ is the Attraction of TZrS, according to HY. In which if we put $\Pi\pi$ for HQ, we ſhall have $\frac{\Pi H\pi R\times 2HY}{NT^3}$, or $\frac{\frac{1}{2}HY\times\Pi H^2\times c}{NT^3}$, for the Attraction required.

IX. It is eaſy to perceive, that if, inſtead of a Circle, the Curve $R\Pi r$ were an Ellipſis, or any other Curve whoſe Axes were but very little different from one another, the foregoing Solution would be ſtill the ſame.

PROBLEM. V.

To find the Attraction which an Elliptical Spheroid KLk (Fig. 5.) *exerts upon a Corpuſcle placed without its Surface at* N, *according to the Direction* CX *perpendicular to* CN.

X. To perform this, we will begin by drawing the Diameter $C\mu\nu$, which biſects the Lines Rr perpendicular to CN; and the Ratio of CH to HY ſhall be call'd n. Then eſteeming the Ellipſis Rr as a Circle, (ſee the foregoing Article) we ſhall have by the Problem aforegoing $\frac{\frac{1}{2}nc\times RH^2\times CH}{NR^3}$ for its Attraction, according to HY; which being multiply'd by the Fluxion of MH, the Fluent of this will be the Attraction of the Segment of the Spheroid RMr.

This Calculation being made, and Nm being ſubſtituted for NR, we ſhall have $\frac{2ncr^5}{5e^4}$ for the Attraction of the Spheroid in N, according to the Direction CX.

PROBLEM VI.

To find the Attraction of a Corpuſcle N, *according to* CX, *towards an Ellipſoid* BNEbe, *compoſed of Strata, the Denſities of which are defined by the Equation* $D=fr^p+gr^q$.

XI. Take the Fluxion of the Quantity $\frac{2cnr^5}{5e^4}$, which expreſſes the Attraction of the homogeneous Ellipſoid KLk, and you will have $\frac{2cnr^4\dot{r}}{e^4}$ for the Attraction of an infinitely little elliptick Orb; which being multiply'd by the Denſity D, gives $\frac{2cnfr^{\overline{4+p}}\dot{r}}{e^4}$ $+\frac{2cgnr^{\overline{4+q}}\dot{r}}{e^4}$, the Fluent of which $\frac{2cfnr^{\overline{5+p}}}{\overline{5+p}\times e^4}$ $+\frac{2cgnr^{\overline{5+q}}}{\overline{5+q}\times e^4}$, is the Attraction of the Spheroid KLk, according to CX. Therefore the total Attraction of the Spheroid BNEbe upon the Corpuſcle N, according to the Direction CX, will be $\frac{2cfne^{\overline{1+p}}}{5+p}+\frac{2cgne^{\overline{1+q}}}{5+q}$.

Now if we have regard to the Smallneſs of the Line Nν, and obſerve how little the Angle νNC will differ from a right one, we may perceive that the Diameter CN contains the ſame Angle with the perpendicular NX in N, as the Diameter CN with the perpendicular at ν; that is to ſay, that the Angle NCν is the ſame as the Angle CNX; ſo that inſtead of n we may take $\frac{CX}{CN}$. Wherefore the foregoing Expreſſion of the Attraction of the Ellipſoid BEbe, acting according to the Direction CX upon a Corpuſcle placed in N, will be $\frac{2cfe^{1+p}}{5+p} \times \frac{CX}{CN} + \frac{2cge^{1+q}}{5+q} \times \frac{CX}{CN}$.

PROBLEM VII.

To find the Direction of the Attraction of a Corpuſcle N *towards the Ellipſoid.*

XII. By the ſecond Problem we ſhall find the Attraction of the Spheroid according to CN to be $\frac{2cfe^{1+p}}{3+p} + \frac{2cge^{1+q}}{3+q}$, by expunging what may be here expunged. Then by taking a fourth proportional to theſe three Quantities, the firſt of which is the Attraction according to CN, the ſecond is that according to CX, and the third is the right line CN; there will ariſe

$$\frac{\frac{fe^{1+p}}{5+p} + \frac{ge^{1+q}}{5+q}}{\frac{fe^{1+p}}{3+p} + \frac{ge^{1+q}}{3+q}} \times CX = CI.$$

Whence we ſhall have NI for the Direction requir'd, of the Attraction of the Corpuſcle N.

XIII. If we ſuppoſe $p=q=o$, that is, if the Spheriod be homogeneous, we ſhall have $CI=\frac{3}{5}CX$; which agrees with what Mr. *Stirling* has found, in that curious Diſſertation he has publiſh'd in the *Philoſophical Tranſactions*, N° 438.

PART II.

The Uſe of the foregoing Problems, in finding the Figure of Spheroids, which revolve about an Axis.

XIV. Let us now ſuppoſe, that the foregoing Spheroid BNE be, (Fig. 5.) which is ſtill compoſed of Beds or *Strata* of different Denſities, revolves about its Axis Bb, and that it is now arrived at its permanent State. It is plain that the Particles of the Fluid, which are upon its Surface, muſt gravitate according to a Direction perpendicular to the Curvature BNE; for without this Condition there could be no *Æquilibrium*.

We ſhall now inquire, whether the Elliptic Figure we have aſcribed to our Spheroids can have this Property, and to produce this Effect, what muſt be the Relation between the Time of Revolution of the Spheroid and the Difference of its Axes.

Let us then put Φ for the centrifugal Force at the Equator, and the centrifugal Force at N will be $\frac{\varphi \times PN}{CE}$, or $\frac{\varphi \times Cx}{2CE \times \alpha}$, becauſe $2PN \times \alpha = Cx$.

By reſolving this centrifugal Force according to the Perpendicular to CN, we ſhall have $\frac{\varphi\times CX}{2\alpha\times CE}$; which being added to $\frac{2cfe^{\overline{1+p}}}{\overline{5+p}}\times\frac{CX}{CN}+\frac{2cge^{\overline{1+q}}}{\overline{5+q}}\times\frac{CX}{CN}$, found by Prob. V. will give the whole Force of the Body N, according to the Direction CX, when the Spheroid is converted about its Axis. But becauſe this Body, by virtue of the Attraction according to CN, and the Force according to CX, ought to have a perpendicular Tendency to the Superficies; we ſhall ſhall have this Analogy, CN. CX : : $\frac{2cfe^{\overline{1+p}}}{\overline{3+p}}+\frac{2cge^{\overline{1+q}}}{\overline{3+q}}.\frac{}{2\alpha}\times\frac{CX}{CE}+\frac{2cfe^{\overline{1+p}}}{\overline{5+p}}\times\frac{CX}{CN}+\frac{2cge^{\overline{1+q}}}{\overline{5+q}}\times\frac{CX}{CN}$. And hence, becauſe CN and CE may be aſſumed as the ſame on this Occaſion, it will be

$$\varphi=\frac{8cfe^{\overline{1+p}}\alpha}{\overline{3+p}\times\overline{5+p}}+\frac{8cge^{\overline{1+q}}\alpha}{\overline{3+q}\times\overline{5+q}}.$$

And as in this Value of the centrifugal Force, no Quantity enters but what will agree to any Point N; we may therefore conclude, that when our ſuppos'd elliptical Spheroid performs its Rotation in a proper Time, ſo that the centrifugal Force at the Equator may be as before; then the centrifugal Force in any other Place N will be ſuch as it ought to be, to cauſe Bodies to gravitate in a perpendicular Direction to the Surface.

The Spheroid being ſuppos'd elliptical, Bodies will gravitate perpendicularly to its Surface.

XV. If

The Expreſſion for the Gravity at any Place on the Spheroid.

XV. If we now conſider, that ED (Fig. 6.) being taken for the centrifugal Force in E, then will MN expreſs the centrifugal Force in N, and conſequently MI will be ſuch a Part of this Force as acts according to NC; we ſhall have $\frac{8cfe^{1+p}\lambda}{\overline{3+p}\times\overline{5+p}}+\frac{8cge^{1+q}\lambda}{\overline{3+q}\times\overline{5+q}}$ to be ſubtracted from the Attraction at N. Hence $\frac{2cfe^{1+p}}{3+p}$ $+\frac{\overline{2p-10}cf\lambda e^{1+p}}{\overline{3+p}\times\overline{5+p}}+\frac{8cf\alpha e^{1+p}}{\overline{3+p}\times\overline{5+p}}+\frac{2cge^{1+q}}{3+q}$ $+\frac{\overline{2q-10}cg\lambda e^{1+q}}{\overline{3+q}\times\overline{5+q}}+\frac{8cg\alpha e^{1+q}}{\overline{8+q}\times\overline{5+q}}$ will be the Gravity at N.

The Gravity at the Equator.

XVI. In this Value making $\lambda=\alpha$, we ſhall have $\frac{2cfe^{1+p}}{3+p}+\frac{\overline{2p-2}cf\alpha e^{1+p}}{\overline{3+p}\times\overline{5+p}}+\frac{2cge^{1+q}}{3+q}$ $+\frac{\overline{2q-2}cg\alpha e^{1+q}}{\overline{3+q}\times\overline{5+q}}$ for the Gravity at the Equator.

XVII. If we ſubtract the Value of the Gravity in N from the Value of the Attraction or Gravity at the Pole, (Art. III.) we ſhall have $\frac{\overline{10-2p}cf\lambda e^{1+p}}{\overline{3+p}\times\overline{5+p}}$ $+\frac{\overline{10-2q}cg\lambda e^{1+q}}{\overline{3+q}\times\overline{5+q}}$. But it is eaſy to perceive, that λ is proportional to the Square of the Sine of the Arc PM, or of the Complement of the Latitude. Whence we may therefore conclude, that the Diminution

nution of the Gravity from the Pole to the Equator is proportional to the Square of the Coſine of the Latitude; or, which is the ſame thing, that *the Augmentation of Gravity from the Equator to the Pole is as the Square of the Sine of the Latitude*, as Sir *Iſaac Newton* has demonſtrated in his Hypotheſis of a homogeneous Spheroid.

XVIII. From the following Calculation it is eaſy to conclude, that Sir *Iſaac*'s Theorem, (*Prin. Math.* L. III. Prop. XX.) which is this, that *the Gravity in any Place within is reciprocally as the Diſtance from the Centre*, cannot obtain here. For we may ſee by the foregoing Expreſſion, that the Gravity in N cannot be to the Gravity in P as 1 to $1+\lambda$, except when $p=q=0$, which happens only in Sir *Iſaac*'s homogeneous Spheroid.

It was for want of conſidering, that this Theorem was demonſtrated by Sir *Iſaac* only in the Caſe of his homogeneous Spheroid, that ſeveral Geometricians have too haſtily concluded, this Theorem might be apply'd to determine the Ratio of the Earth's Axes, and the Lengths of the Pendulum obſerved in two Places of different Latitudes. Dr. *Gregory* is one of thoſe who have fallen into this Miſtake, in his *Elements of Aſtronomy*, Lib. III. Sect. 8. Prop. 52. And in the *Philoſophical Tranſactions*, Nº 432. it is concluded, from the Proportion of Gravity at *Jamaica* to that at *London*, that the Diameter of the Equator muſt exceed the Earth's Axis by $\frac{1}{190}$th Part, which Computation was founded on this 20th Propoſition, Lib. III. of Sir *Iſaac*'s *Principia*, which is true only of his Spheroid.

The Manner of finding the Axes of the Spheroid, the Variation of the Denſities of the Strata *being taken at pleaſure.*

XIX. Let us now ſuppoſe, that the centrifugal Force at the Equator is known by Obſervation, as alſo within the Earth, *&c.* and that it is a certain Part $\frac{1}{m}$ of the Gravity; by Articles XIV. and XVI. we ſhall have this Equation:

$$\frac{2cfe^{1+p}}{3+p}+\frac{\overline{2p}-2cfe^{1+p}\alpha}{\overline{3+p}\times\overline{5+p}}+\frac{2cge^{1+q}}{3+q}+\frac{\overline{2q}-2cge^{1+q}\alpha}{\overline{3+q}\times\overline{5+q}}=\frac{8cfme^{1+q}\alpha}{\overline{3+p}\times\overline{5+p}}+\frac{8cmge^{1+q}\alpha}{\overline{3+q}\times\overline{5+q}}.$$

From hence it will be eaſy to derive the Value of α, becauſe f, g, p, q, will be given, from the Hypotheſis that will be choſen, for the Variation of the Denſity in the internal Parts of the Spheroid.

XX. And if on the contrary α be given, that is, if we know by Obſervation the Ratio of the Axes of the Planet concern'd; then by the foregoing Equation we may perceive, whether we have aſſumed an agreeable Hypotheſis for the Variation of the Denſities: But we cannot preciſely determine what this Hypotheſis muſt be, becauſe there is but one Equation, in which four indeterminate Quantities f, g, p, q, are involved. And indeed there might be many more than four indeterminate Quantities, if we ſhould aſſume more than two Terms in the general Equation of the Denſities $D=fr^p+gr^q+hr^s$, *&c.*

XXI. In order to apply the foregoing Theory to the Earth, it might ſeem at firſt Sight, that by the Aſſiſtance of Obſervations made for meaſuring the Length of of the Pendulum, we might have other Equations, which with the foregoing Equation A, would determine the Coefficients & Exponents now mention'd; but we ſhall ſoon ſee the Impoſſibility of this upon two Accounts: Firſt, There need be only two Obſervations, as to what concerns the Length of the Pendulum. For becauſe by Art. XVII. the Augmentation of the Gravity from the Equator to the Pole is proportional to the Square of the Sine of the Latitude, two Obſervations as much determine the Problem as an infinite Number can do: So that we could have but one other Equation beſides the foregoing. This Equation will be (B) $\frac{\overset{\prime}{p}-p}{p}=\frac{\frac{\overline{5-p}\,f\alpha}{\overline{3+p}\times\overline{5+p}}+\frac{\overline{5-q}\,g\alpha}{\overline{3+q}\times\overline{5+q}}}{\frac{\overline{p-1}\,f\alpha}{\overline{3+p}\times\overline{5+p}}+\frac{f}{3+p}+\frac{g}{3+q}+\frac{\overline{q-1}\,g\alpha}{\overline{3+q}\times\overline{5+q}}}$

The firſt Member of this Equation expreſſes the Gravity at the Equator ſubtracted from the Gravity at the Pole, and divided by the Gravity at the Equator; a Quantity which may be known in Numbers, by determining the Length of the Pendulum at two different Latitudes. The other Member of the Equation is an Expreſſion of the ſame Quantity, as it is deduced by the preceding Calculus.

Secondly, This new Equation B cannot be of any Service in determining the Coefficients and Exponents f, g, p, q, *&c.* For we ſhall now ſhew, that the foregoing Ratio $\frac{\overset{\prime}{p}-p}{p}$ has ſuch an immediate Connexion with α, that one of them being determin'd, the other will neceſſarily be ſo too, independently of the

the Values of f, g, p, q, *&c.* This may deſerve our Attention, and the Proof is thus:

XXII. Becauſe the Ratio of the Gravity to the Centrifugal Force is very great, and is expreſs'd by m, in the Equation A we may reject the third and fourth Terms; by which means the Equation will be reduced to this,

$$\frac{f}{3+p}+\frac{g}{3+q}=\frac{4mf\alpha}{\overline{3+p}\times\overline{5+p}}+\frac{4mg\alpha}{\overline{3+q}\times\overline{5+q}}.$$

And if from this Equation we deduce the Value either of f or g, and ſubſtitute it in the Equation B; (having firſt rejected the firſt and fourth Terms of the Denominator, as in this Caſe may be done) we ſhall have after the Calculation is made, whatever is the Number of Terms in the Equation of the Denſities, $\frac{p'-p}{p}=\frac{10}{4m}-\alpha$, or $\frac{p'-p}{p}=\frac{1}{115}-\alpha$, by putting 288 for m, as has been long known. It is eaſily ſeen from this Equation, that when α is determined, $\frac{p'-p}{p}$ will be ſo too, which was the thing propoſed to be proved.

The Figure of the Spheroid being known, the Augmentation of Gravity from the Equator to the Pole will be known alſo; and ſo vice verſa.

XXIII. But from this Equation there follows a very ſingular Propoſition, and which, in ſome ſort, is contrary to the Sentiments of Sir *Iſaac Newton*, Page 430. of the 3d Edition of his Principles. And this is, that *if by Obſervation it ſhall be diſcover'd, that the Earth is flatter than according to the Spheroid of Sir* Iſaac, *that is, if the Diameter of the Equator exceeds the Axis by more than the* $\frac{1}{230}$ *Part, the*

Gravity will increaſe leſs from the Equator towards the Pole, than according to the Table which he has given for his Spheroid; Prop. XX. of the 3d Book. And on the contrary, *if the Spheroid is not ſo flat, the Gravity will increaſe more from the Equator towards the Pole.*

XXIV. 'Tis thus that Sir *Iſaac Newton* expreſſes himſelf about it, when he relates the Experiments made towards the South, concerning the Diminution of Gravity, which Experiments make it greater than his Theory requires. He affirms, that the Earth is denſer towards the Centre than at the Superficies, and more depreſs'd than his Spheroid requires. But by the foregoing Theory we may eaſily perceive, that if the Denſity of the Earth diminiſhes from the Centre towards the Superficies, the Diminution of Gravity from the Pole towards the Equator will be greater than according to Sir *Iſaac*'s Table; but at the ſame time the Earth will be not ſo much depreſs'd as his Spheroid requires, inſtead of being more ſo, as he affirms. Yet I would not by any means be underſtood to decide againſt Sir *Iſaac*'s Determination, becauſe I cannot be aſſured of his Meaning, when he tells us, that the Denſity of the Earth diminiſhes from the Centre towards the Circumference. He does not explain this, and perhaps inſtead of the Earth's being compos'd of parallel Beds or *Strata*, its Parts may be con-

Et exceſſus longitudinis Penduli Pariſienſis ſupra longitudines Pendulorum iſochronorum in his latitudinibus obſervatas, ſunt paulo majores quam pro Tabula longitudinum Penduli ſuperius computata. Et propterea Terra aliquanto altior eſt ſub æquatore, quam pro ſuperiore calculo, & denſior ad centrum quam in fodinis prope ſuperficiem.

conceived to be otherwiſe arranged and diſpoſed, ſo as that the Propoſition of Sir *Iſaac* ſhall be agreeable to the Truth.

XXV. As to Dr. *Gregory*, who has attempted to comment upon this Paſſage of Sir *Iſaac*, I think I have demonſtrated, that he has committed a Paralogiſm. He ſays (*Element. Aſtronom.* Lib. III. §. VIII. Prop. LII. Schol.) that if the Earth is denſer towards the Centre, or if (for Example) it has a Nucleus of greater Weight than the other Parts, the Diminution of Gravity from the Pole towards the Equator ſhall be greater than if the whole were of the ſame Denſity; and in this he is right. But he is in the wrong (I think) immediately to conclude from thence, that the Earth has a greater Flatneſs. Whence can he conclude this? It can be only from that Propoſition of Sir *Iſaac* which informs us, that Gravity is in a reciprocal Ratio of the Diſtances; becauſe he gave us the Propoſition but the Page before, as a Method for determining the Figure of the Earth. But we are not allow'd to make uſe of this Propoſition in this Caſe, becauſe it has been ſhewn, Art. XVIII. that it can take Place only on the Suppoſition of a homogeneous Spheroid. Therefore, *&c.*

XXVI. It will not be very difficult, without any Regard had to the foregoing Theory, to find the Ratio of the Axes of a Spheroid, which we may ſuppoſe to have a Nucleus at the Centre, of greater Denſity than the reſt of the Planet; and hence we ſhall be eaſily aſſured of Dr. *Gregory*'s Miſtake.

XXVII. Setting aſide all Attraction of the Parts of Matter, if the Action of Gravity is directed towards a Centre, and is in the reciprocal Ratio of the Squares of the Diſtances, the Ratio of the Axes of the Spheroid will then be that of 576 to 577: And the Gravity at the Pole is greater than at the Equator by $\frac{1}{144}$th Part, or thereabouts. Which may be a Confirmation of what is here advanced, eſpecially to ſuch as will not be at the Pains of going through the foregoing Calculations. For we may conſider the Spheroid now mention'd, in which Gravity acts in a reciprocal Ratio of the Squares of the Diſtances, as compoſed of Matter of ſuch Rarity, in reſpect of that at the Centre, that the Gravity is produced only by the Attraction of the Centre or Nucleus.

XXVIII. In the foregoing Calculations, in order to find the Axes of our Spheroids, and to know whether their Figure makes a ſenſible Approach to that of the conical Ellipſis, we have had Recourſe to this Principle, that Gravity ought always to act in a Direction perpendicular to the Surface. Two Reaſons have prevail'd with us to make uſe of this Principle rather than the other, which conſiſts in the Equilibrium of the Columns. The firſt is, becauſe the Calculations founded thereon are more ſimple. The ſecond is, that conſidering the State of the actual Solidity of the Earth, it ſhould ſeem as if this Principle were the more indiſpenſably neceſſary. However, becauſe Sir *Iſaac Newton*, and all the other Philoſophers, who have treated about the Figure of the Earth, have taken it, as it were, at its firſt Formation, at which Time they ſuppoſe it to have been fluid; we ſhall here make the ſame Suppoſition, and we ſhall aſſume no other Ratio for that of the two Axes, than that of the Spheroid, which reſults from a Coincidence of theſe two Principles.

We ſhall begin by inquiring what is the intire Weight of any Column CN, Fig. 7. To do this we muſt reſume the Expreſſion of the Attraction in any Point M of the Column CN; then multiply it by $\dot{r}+\lambda\dot{r}$, and by the Denſity $fr^{p}+gr^{q}$, and afterwards we muſt find the Fluent. Thus we ſhall have

$$\frac{cf^2e^{2+2p}}{\overline{1+p}\times\overline{3+p}}+\frac{cg^2e^{2+2q}}{\overline{1+q}\times\overline{3+q}}+\frac{2cfge^{2+p+q}}{\overline{2+p+q}\times\overline{3+p}}$$

$$+\frac{2cfge^{2+p+q}}{\overline{2+p+q}\times\overline{3+q}}+\frac{4cf^2\alpha e^{2+2p}}{\overline{1+p}\times\overline{3+p}\times\overline{5+p}}$$

$$+\frac{4cg^2\alpha e^{2+2q}}{\overline{1+q}\times\overline{3+q}\times\overline{5+q}}+\frac{8cfg\alpha e^{2+p+q}}{\overline{2+p+q}\times\overline{3+p}\times\overline{5+q}}$$

$$+\frac{8cfg\alpha e^{2+p+q}}{\overline{2+p+q}\times\overline{3+q}\times\overline{5+q}}+\frac{\overline{4+2p}\,cf^2\lambda e^{2+2p}}{\overline{1+p}\times\overline{3+p}\times\overline{5+p}}$$

$$+\frac{\overline{4+2q}\,cg^2\lambda e^{2+q}}{\overline{3+q}\times\overline{5+q}\times\overline{1+p}}+\frac{\overline{8+4p}\,cgf\lambda e^{2+p+q}}{\overline{2+p+q}\times\overline{3+p}\times\overline{5+p}}$$

$$+\frac{\overline{8+4q}\,cfg\lambda e^{2+p+q}}{\overline{2+p+q}\times\overline{3+q}\times\overline{5+q}}$$

for the total Gravity of any Column CN, having Regard only to the Attraction.

XXIX. If in this Expreſſion we make $\lambda=0$, we ſhall have the Gravity of the Column at the Pole.

XXX. And if we make $\lambda=\alpha$, we ſhall have the Aggregate of the Attractions of the Column at the Equator.

XXXI. Now becauſe the Column CN is *in Æquilibrio* with the Column CB; it follows from thence, that if we ſubtract the Weight of the Column CB from the Aggregate of the Attractions of the Column CN, the Reſidue muſt be equal to the Sum of the centrifugal

Forces of the Column CN. Now to endue our Spheroids with this Property, we will reſume the Expreſſion of the centrifugal Force in E, which we found Art. XIV. which will give $\left(\frac{8cfe^{\overline{1+p}}\lambda}{\overline{3+p}\times\overline{5+p}}+\frac{8cge^{\overline{1+p}}\lambda}{\overline{3+q}\times\overline{5+q}}\right)\frac{r}{e}$, for that Part of the centrifugal Force which acts according to CM, in any Place M, by expunging the Terms in which $\alpha\alpha$ would be found. This Value being multiply'd by $\dot{r}$, and by the Denſity, will give (when we have taken the Fluent) $\frac{8cf^2e^{\overline{2+2p}}\lambda}{\overline{2+p}\times\overline{3+p}\times\overline{5+p}}$

$$+\frac{8cfge^{\overline{2+p+q}}\lambda}{\overline{2+p}\times\overline{3+q}\times\overline{5+q}}+\frac{8cfge^{\overline{2+p+q}}\lambda}{\overline{2+q}\times\overline{3+p}\times\overline{5+p}}$$

$+\frac{8cg^2e^{\overline{2+2q}}\lambda}{\overline{2+q}\times\overline{3+q}\times\overline{5+q}}$ for the Sum of the centrifugal Forces of the Column CN, ſtill expunging thoſe Terms in which either $\alpha\alpha$ or $\lambda\lambda$ are found.

Then making this Expreſſion equal to

$$\frac{\overline{4+2p}\,cf^2e^{\overline{2+2p}}\lambda}{\overline{1+p}\times\overline{3+p}\times\overline{5+p}}+\frac{\overline{8+4p}\,cfge^{\overline{2+p+q}}\lambda}{\overline{2+p+q}\times\overline{3+p}\times\overline{5+p}}$$

$$+\frac{\overline{8+4q}\,cfge^{\overline{2+p+q}}\lambda}{\overline{2+p+q}\times\overline{3+q}\times\overline{5+q}}+\frac{\overline{4+2q}\,cg^2e^{\overline{2+2q}}\lambda}{\overline{1+q}\times\overline{3+q}\times\overline{5+q}},$$

which is the Difference of the Weight of the Column at the Pole CB, from the Sum of the Attractions of the Column CN, we ſhall have the Equation

$$\frac{ppff}{\overline{1+p}\times\overline{2+p}\times\overline{3+p}\times\overline{5+p}}+\frac{2pqfg}{\overline{2+p+q}\times\overline{3+p}\times\overline{5+p}\times\overline{2+q}}$$

$$+\frac{2pqfg}{\overline{2+p+q}\times\overline{3+q}\times\overline{5+q}\times\overline{2+p}}+\frac{qqgg}{\overline{1+q}\times\overline{2+q}\times\overline{3+q}\times\overline{5+q}}=0,$$

where we have put $e=1$, for the greater Simplicity of Calculation.

XXXII. This Equation informs us, that when out of all the infinite Varieties, which will be ſupply'd by the Equation of the Denſities $D = fr^p + gr^q + hr^s$, *&c.* we ſhall have taken at Pleaſure all the Coefficients, and all the Exponents, one only excepted; if this laſt is ſuch in reſpect of the others, that it may fulfil the Conditions of the foregoing Equation, the Spheroid, being ſuppoſed in a State of Fluidity, will be *in Æquilibrio*, becauſe it will unite as well the Principle of a perpendicular Tendency to the Surface, as that of an Equipoiſe of the ſeveral Columns.

Determination of ſuch Spheroids, as make the Principle of the Equilibrium of the Columns, and that of Gravity perpendicular to the Surface, to coincide with each other.

XXXIII. Before I conclude this Paper, I ſhall make a few Reflections on the Principles we have now made uſe of, for determining the Figure of a Spheroid revolving about its Axe.

The firſt Principle which, after Mr. *Huygens*, we have had Recourſe to, and which conſiſts in making Bodies gravitate perpendicularly to the Surface, ſeems to me of abſolute Neceſſity. For if there were never ſo little Water upon the Surface of the Earth, it could not be at Reſt, if it had a Tendency any how inclined to the Surface.

The ſecond Principle, made uſe of by Sir *Iſaac Newton*, and which conſiſts in an Equilibrium of the Columns CE, CN, CP, could be thought neceſſary (I think) only for theſe two Reaſons: The firſt is that which is uſually aſſign'd, that at the firſt Formation of the Earth, it was probably in a State of perfect Fluidity; in which caſe it muſt acquire ſuch a Figure, as will reſult from the Equilibrium of the Columns, and from the Gravitation acting perpendicularly to the Surface. Indeed though this Reaſon has a Degree of Plauſibility, yet there are many who think it to be of ſmall Force. Perhaps, ſay they, the Earth has never been in this fluid Condition.

The ſecond Reaſon, which I believe will have a greater Weight with every Body, is this. Conſidering the Earth as it is at preſent, and without carrying our Thoughts ſo far back as to its Formation, if the Ocean, which is now upon its Surface, has any conſiderable Depth, and if its Parts preſerve a Communication with each other, from Region to Region, by ſubterraneous Canals; it can only keep an Equilibrium by this Means, becauſe its Superficies is the ſame as it would have, were the whole a Fluid.

XXXIV. This ſecond Reaſon has ſuggeſted a Reflexion to my Mind, concerning the Equipoiſe of the Columns now calculated, Art. XXXI. and XXXII. Let us firſt ſuppoſe, that the Earth is our fluid Spheroid, compoſed of Beds of different Denſities; and that afterwards this Fluid hardens into a Solid, ſo that the different Beds or *Strata*, of which it is made up, are of no other Uſe but to cauſe a Gravity by their Attractions. Then let us ſuppoſe, that the Seas and great Waters about the Earth have a Communication with each other, by means of ſome ſubterraneous Canals. As the Waters of the Sea, which unite with one another, are probably homogeneous, the foregoing Calculation, wherein we have conſider'd the Spheroid as a Fluid, can no longer take Place, becauſe we have there ſuppoſed, that the Fluid contain'd in the Canal BCN is of a Denſity, that varies from the

Centre to the Circumference. From hence it ſeems to me, we muſt undertake the Computation of the Equilibrium of the Columns after another Manner, thus:

We muſt examine whether two Canals, as CN and BC, which are fill'd with a homogeneous Fluid, will be *in Æquilibrio*, all the other Parts of the Spheroid continuing as above.

XXXV. To do this, we will begin with finding the Gravity of any Column CN, (Fig. 7.) ariſing from Attraction alone. Firſt, then, we muſt reſume the Expreſſion of the Attraction in any Point M, Art. VII. Then we muſt multiply it by $\dot{r}+\lambda\dot{r}$, which will give $\frac{2cfr^{\overline{1+p}}\dot{r}}{3+p}+\frac{\overline{8+4p}\,cf\lambda r^{\overline{1+p}}\dot{r}}{\overline{3+p}\times\overline{5+p}}+\frac{8cf\alpha r^{\overline{1+p}}\dot{r}}{\overline{3+p}\times\overline{5+p}}$ $+\frac{2cgr^{\overline{1+q}}\dot{r}}{3+q}$, *&c.* And taking the Fluent of this Quantity, we ſhall have $\frac{2cfe^{\overline{2+p}}}{\overline{3+p}\times\overline{2+p}}+\frac{4cf\lambda e^{\overline{2+p}}}{\overline{3+p}\times\overline{5+p}}$ $+\frac{8cf\alpha e^{\overline{2+p}}}{\overline{2+p}\times\overline{3+p}\times\overline{5+p}}+\frac{2cge^{\overline{2+q}}}{\overline{3+q}\times\overline{2+q}}$, *&c.* for the Gravity of the whole Column CN.

XXXVI. If in this Value we make $\lambda=0$, we ſhall have the Gravity of the Column at the Pole.

XXXVII. And if we ſubtract the Gravity of the Column at the Pole from the whole Sum of the Attractions of the Column CN, we ſhall have $\frac{4cf\lambda e^{\overline{2+p}}}{\overline{3+p}\times\overline{5+p}}+\frac{4cge^{\overline{2+q}}\lambda}{\overline{3+q}\times\overline{5+q}}$, which muſt be equal to the Sum of the centrifugal Forces of the Column CN, in order that the Columns CB and CN may be *in Æquilibrio.*

But we ſhall find this really to obtain, if we reſume the Quantity $\left(\frac{8cfe^{1+p}}{\overline{3+p}\times\overline{5+p}}+\frac{8cge^{1+q}\lambda}{\overline{3+q}\times\overline{5+q}}\right)\frac{r}{e}$, which expreſſes (Art. XXXI.) that Part of the centrifugal Force in M, which acts according to CM. Then multiplying this Expreſſion by $\dot{r}$, and ſeeking the Fluent, we ſhall have $\frac{4cfe^{2+p}\lambda}{\overline{3+p}\times\overline{5+p}}+\frac{4cge^{2+q}\lambda}{\overline{3+q}\times\overline{5+q}}$ for the Aggregate of the centrifugal Forces of the Column CN. And this being the ſame as the foregoing, ſhews, that the Columns CB and CN are *in Æquilibrio*, ſuppoſing them to be homogeneous; nor are we here obliged, as in Art. XXXII. where we conſider them as heterogeneous, to ſuppoſe the Coefficients f, p, *&c.* to have any certain Relation among one another.

XXXVIII. Perhaps it may be urged, that the foregoing Calculus agrees only to a Canal, as BCN, which paſſes through the Centre; and that we ought to prove, in the ſame manner, that the Water included in any other Canal pqr would obſerve an *Æquilibrium.* But it appears to me, that this Property may be derived from the former: For it follows from the foregoing Calculation, that if we might be allow'd to make this Hypotheſis, *viz.* That independently of the Attraction of any Matter, the Gravity at any Diſtance CN from the Centre, (ſee Fig. 7.) would be proportional to $\frac{2cfe^{1+p}}{3+p}+\frac{\overline{2p-2cf\lambda}\,e^{1+p}}{\overline{3+p}\times\overline{5+p}}+\frac{8cf\alpha e^{1+p}}{\overline{3+p}\times\overline{5+p}}$, *&c.* it is plain from thence, that a Maſs of the homogeneous Fluid, which ſhould turn about the Axis C B, would aſſume the ſame Form as that of our heterogeneous Fluids. But if this Spheroid ſhould then put on a fix'd State, except only ſome Canal p q r, the Water in this Canal would be *in Æquilibrio*; for without this, the Spheroid could not be eſteem'd as having arrived to its fix'd State. But this Suppoſition comes to the ſame as that of our heterogeneous Spheroid, compoſed of elliptical Beds, in which ſhould be found a Canal p q r of a homogeneous Fluid; provided that the Space, which this Canal poſſeſſes in the Globe, be not of ſo large an Extent, as to change the Law of Attraction.

It might now be thought ſeaſonable to give Examples, for illuſtrating the foregoing Theory; but they are ſo eaſy to be produced, aftet what is already done, that I ſhall leave them to the inquiſitive Reader, having perhaps exceeded the Limits, within which this Diſcourſe ſhould be confined. Therefore I ſhall only add the few Obſervations following.

The only three Planets, in which we can be aſſured of Gravitation, and the centrifugal Force, are the Sun, Jupiter, and the Earth. As to the Sun, the centrifugal Force is there ſo ſmall, in reſpect of its Gravity, that his Poles muſt be very little depreſs'd, ſo that we cannot be ſenſible of it by Obſervation. Then as to Jupiter, Obſervations make him ſomething leſs flat than according to Sir *Iſaac Newton*; that is to ſay, than if he were compoſed of Matter of an uniform Denſity. Therefore by the foregoing Theory, he muſt be a little more denſe towards the Centre, than at the Parts near the Superficies. We might make a thouſand Hypotheſes about the Manner of diſtributing the Inequality of Denſity, proceeding from the Centre towards the Circumference, which would all agree with the Figure obſerved, and which are very eaſy to calculate by the Principles here laid down.

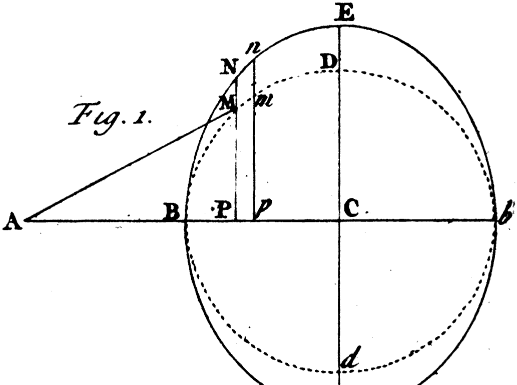
As to what concerns the Earth, I ſhall wait till we receive the Obſervations which muſt have been lately made in *Peru*; that by comparing thoſe with what Obſervations we have made under the arctick Circle, and with thoſe of Mr. *Picart* in *France*, we may have the true Difference of the Earth's Diameters at the Equator and at the Poles. Then our Theory may be apply'd, to determine whether the Earth is more or leſs denſe at the central Parts than at the Surface, or whether it be every-where of an uniform Denſity, as it ought to be, if (without admitting very groſs Errors in the Obſervations) it may be concluded, that the Earth is really the Spheroid of Sir *Iſaac Newton*; and this Caſe would be the ſimpleſt and the moſt natural of all.

I am here obliged to acknowledge, that if the Obſervations we have made in the North may be rely'd upon, and if we muſt admit as inconteſtable as well the Meaſure of a Degree as the Length of the Pendulum, the foregoing Theory could not be reconciled to the *Phænomena*. For it follows from our Obſervations, that the Diameter of the Equator muſt exceed the Earth's Axis by more than $\frac{1}{130}$ Part: And that the Gravity at the Pole muſt be greater than that at the Equator by more than $\frac{1}{130}$ Part likewiſe; which will by no means agree with what we have deduced in Art. XXIII.

As to what concerns the Meaſure of Gravity in *Lapland*, as being not ſo liable to Error as the meaſuring

ſuring a Degree; the Earth may be not quite ſo flat as Sir *Iſaac*'s Spheroid requires. By the Table of the Length of the Pendulum, exhibited in the Treatiſe concerning the Figure of the Earth, publiſh'd this Year by Mr. *de Maupertuis*, and by Art. XXII. of the preſent Diſcourſe, the Earth may be more elevated at the Equator than at the Pole by the $\frac{1}{266}$ Part, or thereabouts. After the true Quantity of the Earth's flatneſs ſhall be fully ſettled, if it ſhould be found to have this Figure, I ſhould be apt to think it is a little more denſe at the Centre than towards the Superficies. But if on the contrary we ſhould be well aſcertain'd, that the Earth is raiſed higher at the Equator than at the Pole, by above the $\frac{1}{230}$ Part; and if, for any ſufficient Reaſon, we may ſomething ſhorten the Length of the Pendulum that beats Seconds in the North; there would be ſome grounds to allow, that the Earth is not ſo denſe at the central Regions as at thoſe near the Surface. But if it ſhall happen, that we can neither diminiſh the Length of the Pendulum, nor the Exceſs of the equatorial Diameter above the Axe; I muſt then give up my Hypotheſis. Yet I ſhall think it may be of ſome Uſe to have thus diſcuſs'd it, becauſe poſſibly no one would have imagined what might have been the Reſult of it. It appears that even Sir *Iſaac Newton* was of Opinion, that it was neceſſary the Earth ſhould be more denſe towards the Centre, in order to be ſo much the flatter at the Poles: And that it follow'd from this greater Flatneſs, that Gravity increaſed ſo much the more from the Equator towards the Pole.

Philoſoph. Transact. N^o. 449. TAB. I.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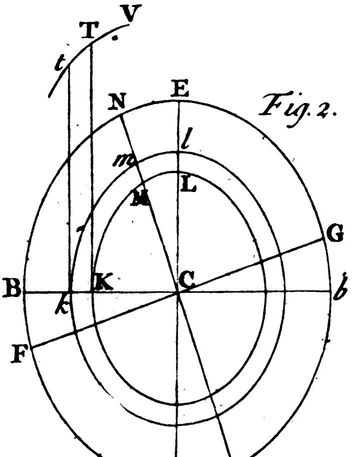
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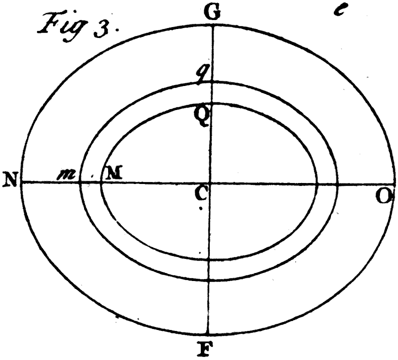
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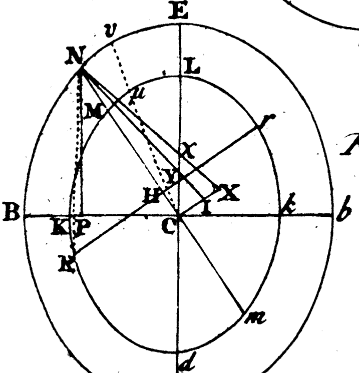
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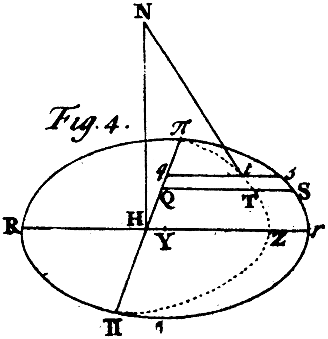
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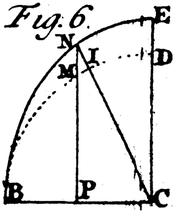
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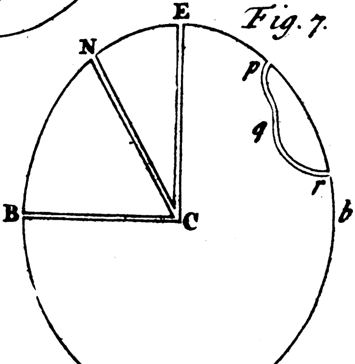
Fig. 7.

J. Mynde sc.